IMPROVED SCS-CN MODEL INCORPORATING STORM INTENSITY FOR RUNOFF ESTIMATION

Nand Kishore Sharma[1], Surendra Kumar Mishra[1], Ashish Pandey[1], Ravindra Kumar Verma[1], Sangeeta Verma[2]

[1]Indian Institute of Technology, Roorkee, India; [2]National Institute of Hydrology, Roorkee, India

ravindra.wt@sric.iitr.ac.in, ravindraverma.nitie@gmail.com

Abstract. The Soil Conservation Service Curve Number (SCS-CN) methodology is the most globally recognized and practiced empirical model for estimation of direct surface runoff from rainfall events, largely due to its simplicity, ease of use, and accounting major runoff producing watershed characteristics. This method (designated as M1) and its explicit form (designated as M2) was originally developed for runoff estimation in small agriculture watersheds of US, now it is also applicable for other land uses. Like other hydrological or hydro-climatic methods, it also has some limitations. Therefore, this paper aims to account for one of the critical limitations, viz., storm duration/intensity and develop an improved SCS-CN model (designated as M3 for general form and M4 for a specific form) for more accurate runoff estimation. The Generalized Reduced Gradient (GRG) non-linear method is also used in this study to optimize the SCS-CN-improved model's parameters. Furthermore, sensitivity analysis is also carried out of the M3 model both analytically and numerically. Sensitivity results show that *P* is the most sensitive variable, whereas *r* is the least sensitive. Finally, all models (M1 through M4) are applied to the rainfall-runoff dataset derived from 45 watersheds of the USDA-ARS. Furthermore, the performance evaluation of all models based on Root Mean Square Error (*RMSE*), Nash Sutcliffe efficiency (*NSE*) (%), Mean absolute error (*MAE*), and *RMSE*-observations standard deviation ratio (*RSR*) revealed the M3 to have performed quite better than all other models in almost all 45 studied watersheds. Overall, based on performance measures, the models' performance from best to worst can be ranked as M3 > M1 > M4 > M2.

Keywords: direct surface runoff, Soil Conservation Service-Curve Number, storm duration, USDA-ARS watersheds.

1. Introduction

The SCS-CN methodology has been utilized by numerous researchers for runoff estimation in agriculture watersheds worldwide since its inception in 1954 [1-3]. Although the recent years have seen the increased interest to the model in different scientific fields, viz., sediment yields, water quality, rain-water harvesting, impacts of forest fire on runoff response and e-flow estimation, its application potential is not yet fully explored. As a result, it has been a subject of intense and extensive exploration for its formation, rationality, applicability and extendibility, physical significance, and so on soon after it came into being. Besides others, the method still inherits a major structural inconsistency associated with the curve number, storm duration, storm intensity, potential maximum retention as it results in abnormality in the description of watershed behavior, as complacent, standard, and violent, and runoff estimation based on the existing SCS-CN method [4; 5].

Despite of this, it is widely accepted among the scientific community, thus several modifications have been explored to forecast runoff in various land uses and climatic conditions, viz., temperate, tropical, semi-arid climate. As a result, extensive research works have been reported on the benefits of incorporating runoff affecting parameters and/or factors, viz., area-specific or representative *CN*, initial abstraction (I_a), region-specific values of λ, antecedent soil moisture (M) variability, potential maximum retention (S), slope adjustment, and remotely sensed and modelled products (e.g., evapotranspiration, rainfall, soil moisture). In [6] the author revisited the method using the entropy theory and provided insights into the method's structure. Later, Verma et al. [7] emphasized chronological evolution of SCS-CN models with their advantages and limitations. They indicated that only a few attempts have been made in the recent past to determine the impact of storm duration on direct surface runoff. In [8] the authors highlighted that an event of low rainfall intensity or high storm duration produces low runoff, leading to a lower value of *CN*, and *vice versa*. [5] also emphasized that rainfall intensity and its spatial and temporal variation affect the runoff generation process. They also described that during storm events of extreme rainfall intensity, the permeable part of the watershed may partly contribute to the runoff production. In [9] it is reported that a watershed with burned conditions generates more runoff due to the change in hydraulic conductivities. Ara and Zakwan also described that the watershed produces high runoff during the period of high rainfall amount [10]. Because the value of *CN* mainly depends on watershed characteristics before the rain, it is also affected by the rain duration besides others, such as antecedent moisture *M*. Although, previous studies indicate that the investigation in developing a mathematical structure by incorporating storm duration and duration-dependent *S* (or *CN*) is still in a premature stage and thus has space for further improvement. Therefore, the main objective of this study is to develop an improved SCS-CN model to be used as a runoff prediction tool. In order to test its efficiency, the following steps are also included: (i) sensitivity analysis prior to model development, (ii) applied on large number of watersheds, e.g., 45 USDA-ARS, and (iii) performance evaluation of the developed models and comparison with existing models using standard statistical indicators.

2. Model development

2.1. Original SCS-CN method

The original method was based on the water balance equation and two proportional hypotheses: the proportional equality hypothesis and linear relationship between I_a and S. These are mathematically expressed as equations 1, 2, and 3, respectively:

$$P = F + I_a + Q, \tag{1}$$

$$\frac{Q}{P - I_a} = \frac{F}{S}, \tag{2}$$

$$I_a = \lambda S. \tag{3}$$

By a combination of equations (1) and (2), a general form of the method is obtained, expressed mathematically,

$$Q = \frac{(P - \lambda S)^2}{P + (1 - \lambda)S}. \tag{4}$$

Equation 4 is based on the fact that runoff begins only after the initial abstraction I_a is satisfied, which means $P \leq I_a$; otherwise, $Q = 0$.

For practical application in US watershed λ was assumed to be 0.2, So, by substituting the assumption into equation 4 gives

$$Q = \begin{cases} \dfrac{(P - 0.2S)^2}{P + 0.8S} \text{ for } P > 0.2S \\ 0 \ \text{ for } P \leq 0.2S \end{cases}. \tag{5}$$

Equations (4) and (5) are referred as models M1 and M2, respectively.

2.2. SCS-CN model incorporating storm duration

In the present study, a modified SCS-CN method including rain duration or its intensity is chosen for comparing with the original method. In fact, both rainfall intensity and its duration for a given rainfall event are important factors in the rainfall-runoff generation process. High rainfall intensity generates high runoff for a given amount of rainfall and *vice versa* only when the rainfall intensity is larger than the infiltration rate of the soil. On the other hand, even a mild intensity rainfall lasting for longer duration may yield a considerable amount of runoff. To account for this effect in modelling, rainfall (P) is adjusted (P_{ad}) with respect to mean rainfall duration, which may represent a characteristic of the watershed, as follows [8]:

$$P_{ad} = P\left(\frac{T}{T_m}\right)^r, \tag{6}$$

where P_{ad} – adjusted rainfall;
T – rain duration;
T_m – mean rain duration;
P – total rainfall depth;
r – exponent.

Now, putting the value of the adjusted rainfall (P_{ad}) in Eq. 4 yields

$$Q = \begin{cases} \dfrac{\left(\left(P\left(\dfrac{T}{T_m}\right)^r - \lambda S\right)\right)^2}{P\left(\dfrac{T}{T_m}\right)^r + (1-\lambda)S}, \text{for } P\left(\dfrac{T}{T_m}\right)^r > \lambda S \\ 0, \text{ for } P\left(\dfrac{T}{T_m}\right)^r \le \lambda S \end{cases}. \quad (7)$$

During original development of the SCS-CN method, λ was assumed to be equal to 0.2. Substituting it into equation 7 gives

$$Q = \begin{cases} \dfrac{\left(\left(P\left(\dfrac{T}{T_m}\right)^r - 0.2S\right)\right)^2}{P\left(\dfrac{T}{T_m}\right)^r + 0.8S}, \text{for } P\left(\dfrac{T}{T_m}\right)^r > 0.2S \\ 0, \text{ for } P\left(\dfrac{T}{T_m}\right)^r \le 0.2S \end{cases}. \quad (8)$$

These above models represented by Eqs. 7 and 8 are referred as M3 and M4, respectively, in the forthcoming text.

3. Study area and hydro-meteorological data

The study area consists of 45 watersheds of the United States, and their locations are shown in Fig. 1. Their size varies from 0.3 to 1773 ha. The *P-Q* datasets have been taken from the USDA-ARS Water Database, which is available at http://www.ars.usda.gov/arsdb.html and http://hydrolab.arsusda.gov/arswater.html.

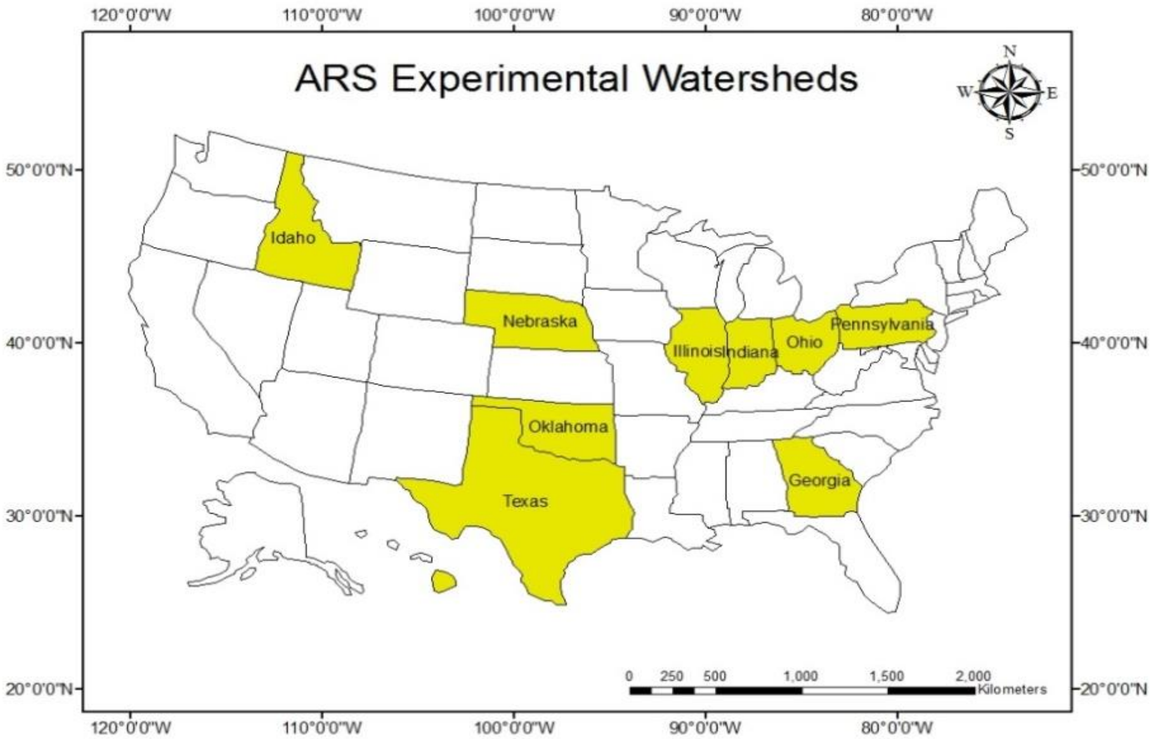


Fig. 1. **Location of study area**

4. Model performance evaluation

4.1. Evaluation indicators

In the present study, the following four widely popular goodness-of-fit indicators are used:

$$RSME = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(Q_{Obs} - Q_{Comp}\right)_i^2}\,, \tag{9}$$

$$RSR = \frac{\sqrt{\sum_{i=1}^{N}\left(Q_{obs} - Q_{comp}\right)_i^2}}{\sqrt{\sum_{i=1}^{N}\left(Q_{obs} - \overline{Q}_{obs}\right)_i^2}}\,, \tag{10}$$

$$NSE = \left[1 - \frac{\sum_{i=1}^{N}\left(Q_{Obs} - Q_{Comp}\right)_i^2}{\sum_{i=1}^{N}\left(Q_{Obs} - \overline{Q}_{Obs}\right)_i^2}\right] * 100\,, \tag{11}$$

$$MAE = \frac{\sum_{i=0}^{n}\left\|Q_{abs} - Q_{comp}\right\|}{n}\,, \tag{12}$$

where Q_{Obs} – observed storm runoff;
Q_{Comp} – computed runoff;
$\overline{Q}_{Obs}$ – mean of observed runoff;
$\overline{Q}_{Comp}$ – mean of computed runoff values in a watershed;
N – total number of rainfall-runoff events;
i – integer varying from 1 to N.

4.2. Parameter estimation

In this study, Generalized Reduced Gradient (GRG) a non-linear method has been used to optimize the SCS-CN-inspired model's parameters, because it is a simple, robust and trustworthy approach to model difficult non-linear systems [11]. Some researcher's, viz., [12-15] successfully used it to estimate the parameters of non-linear Muskingum models for flood routing, intensity-duration-frequency curves, infiltration and soil moisture equations, respectively. The obtained parameter statistics is given in Table 1.

Table 1

Parameter statistics resulting from model application to data of 45 watersheds

Model	Var.	Min.	Max.	Mean	Median	LB	UB
M1	λ	0.00	0.82	0.06	0.00	0.02	0.10
	S	32.31	995.86	155.62	111.13	109.95	201.29
M2	S	35.61	165.29	71.80	67.05	63.19	80.41
M3	λ	0.00	0.27	0.01	0.00	0.00	0.03
	r	0.00	30.16	0.84	0.15	-0.47	2.15
	So	51.57	1306.24	231.80	158.17	161.90	301.70
M4	r	0.00	0.45	0.09	0.07	0.06	0.11
	S	35.89	232.78	81.51	70.46	69.75	93.27

5. Results and discussion

5.1. Sensitivity analysis of the proposed model

The equation of the runoff coefficient ($C = Q/P$) can be obtained as

$$C=\frac{\left(\left(\frac{T}{T_m}\right)^r-\frac{\lambda S}{P}\right)^2}{\left(\left(\frac{T}{T_m}\right)^r+\frac{S}{P}(1-\lambda)\right)}\,. \tag{13}$$

The relative change in *C* to that in *P* is given by

$$\frac{dC/C}{dP/P}=\frac{S\left(P\left(\frac{T}{T_m}\right)^r-\lambda S\right)\left(2\lambda\left(P\left(\frac{T}{T_m}\right)^r-(\lambda-1)S\right)+(\lambda-1)\left(\lambda S-P\left(\frac{T}{T_m}\right)^r\right)\right)}{\left[P\left(\frac{T}{T_m}\right)^r+(\lambda-1)S\right]\left(P\left(\frac{T}{T_m}\right)^r-\lambda S\right)^2}\,. \tag{14}$$

Similarly, the relative change in *C* to that in *S*, *λ*, *T* and *r* is given by Eqs. (15), (16), (17) and (18), respectively.

$$\frac{dC/C}{dS/S}=\frac{S\left(\lambda S-P\left(\frac{T}{T_m}\right)^r\right)\left(2\lambda\left(P\left(\frac{T}{T_m}\right)^r-(\lambda-1)S\right)+(\lambda-1)\left(\lambda S-P\left(\frac{T}{T_m}\right)^r\right)\right)}{\left[P\left(\frac{T}{T_m}\right)^r+(1-\lambda)S\right]\left(P\left(\frac{T}{T_m}\right)^r-\lambda S\right)^2}, \tag{15}$$

$$\frac{dC/C}{d\lambda/\lambda}=\frac{\lambda S\left(\lambda S-P\left(\frac{T}{T_m}\right)^r\right)\left(P\left(\frac{T}{T_m}\right)^r+\lambda S-2S(\lambda-1)\right)}{\left[P\left(\frac{T}{T_m}\right)^r+(1-\lambda)S\right]\left(P\left(\frac{T}{T_m}\right)^r-\lambda S\right)^2}, \tag{16}$$

$$\frac{dC/C}{dT/T}=\frac{\Pr T\left(\frac{T}{T_m}\right)^{r-1}\left(\lambda S-P\left(\frac{T}{T_m}\right)^r\right)\left(-P\left(\frac{T}{T_m}\right)^{r-1}+\lambda S-2S\right)}{T_m\left[P\left(\frac{T}{T_m}\right)^r+(1-\lambda)S\right]\left(P\left(\frac{T}{T_m}\right)^r-\lambda S\right)^2}, \tag{17}$$

$$\frac{dC/C}{dr/r}=\frac{\Pr\left(\frac{T}{T_m}\right)^r\left(\lambda S-P\left(\frac{T}{T_m}\right)^r\right)\left(-\lambda S-P\left(\frac{T}{T_m}\right)^{r-1}+2S(\lambda-1)Ln\left(\frac{T}{Tm}\right)\right)}{\left[P\left(\frac{T}{T_m}\right)^r+(1-\lambda)S\right]\left(P\left(\frac{T}{T_m}\right)^r-\lambda S\right)^2}. \tag{18}$$

Fig. 2 and 3 depict the sensitivity of *C* to *P*, *S*, *λ*, *T* and *r*. In these Figures, $(\partial C/C)/(\partial P/P)$, $(\partial C/C)/(\partial S/S)$, $(\partial C/C)/(\partial\lambda/\lambda)$, $(\partial C/C)/(\partial T/T)$, and $(\partial C/C)/(\partial r/r)$ represent relative change in *C* with respect to partial change in *P*, *S*, *λ*, *T* and *r*, respectively.

It is seen from the Fig. 2 (a) that this ratio decreases exponentially as *P* increases, and further decreases gradually as the rainfall amount increases. Since the rate of increase in *C* with incremental change in *P* decreases as *P* increases, $\delta C/\delta P$ will positively decrease with *P*. The variation seems less for different values of *T* and *r*. Fig. 2 (b) shows the relative change in *C* with respect to partial change in *S*, which indicates that the ratio $((\delta C/C)/(\delta S/S))$ is negative at any *P*-value. However, the absolute value of the ratio increases exponentially with increase in rainfall *P*. For very high rainfall when the soil is near saturation the variation reduces. It is also found that sensitivity of *C* to *S* decreases with increase in *P* for all values of *CN*, λ, *T* and *r*. The sensitivity of *C* to *S* remains unchanged for different values of *T*.

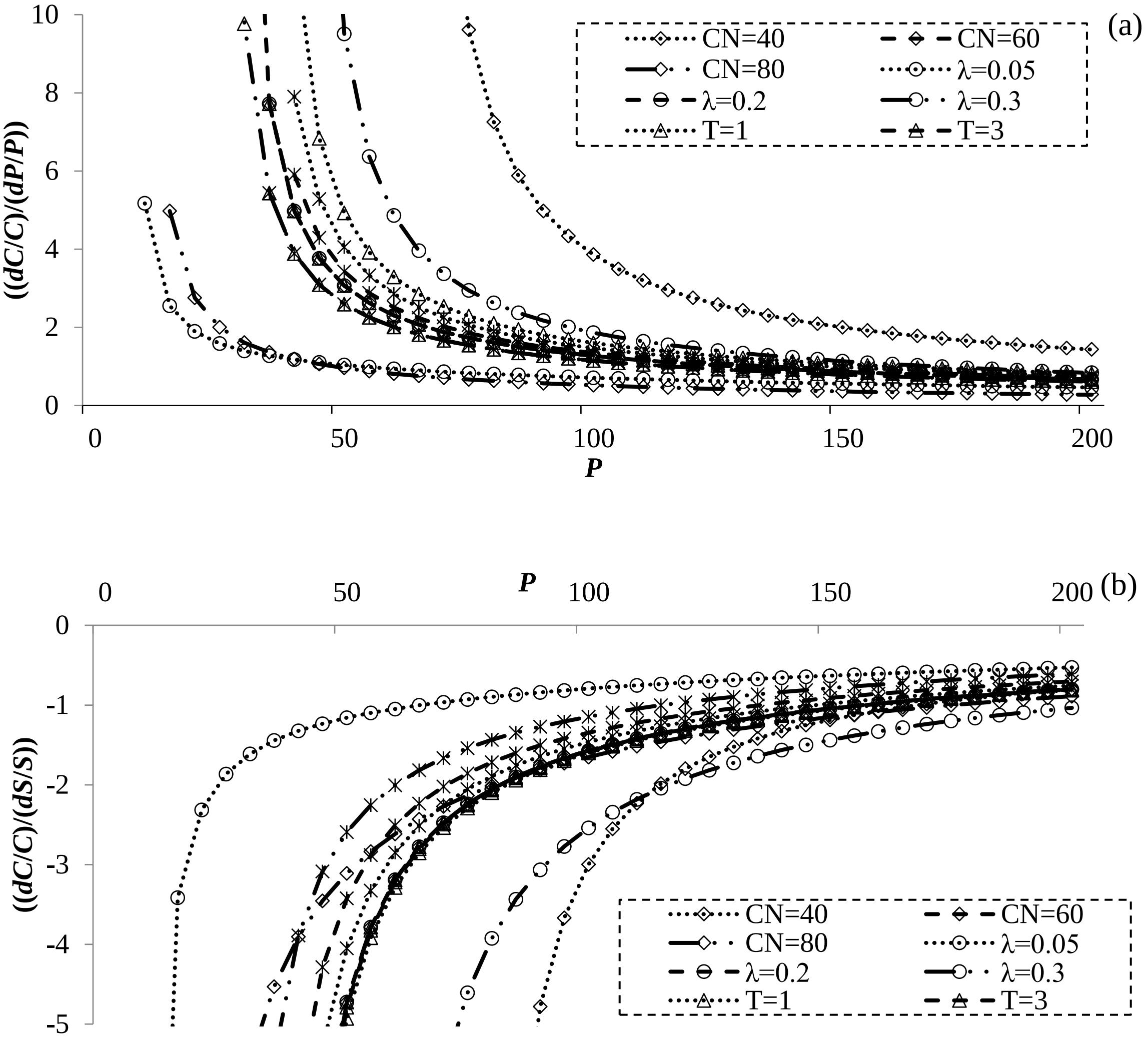


Fig. 2. **Sensitivity analysis of the proposed model w.r.t. its parameters:** relative change in *C* with respect to partial change in *S*

Similarly, Fig. 3 (a), (b) and (c) shows the relative change in *C* with respect to partial change in λ, *T* and *r*, respectively. It can be seen that the ratio $((\delta C/C)/(\delta\lambda/\lambda))$ is negative at any *P*-value and shows a similar trend as the parameter S, whereas the ratio $(\partial C/C)/(\partial T/T)$, and $(\partial C/C)/(\partial r/r)$ is positive at any *P*-value and shows a similar trend as the parameter *P*.

From the results of the sensitivity analysis of the model (M3), it is clear that *P*, *T* and *r* are positively sensitive, whereas other parameters *S* and λ are negatively sensitive. Thus, *P* is the most sensitive variable, whereas *r* is the least sensitive.

5.2. Performance evaluation of all models

The M3 model and its explicit form M4 model replace rainfall (*P*) in the original model with adjusted rainfall (P_{ad}) for improved runoff prediction in a watershed. Model M3 allows variation in λ, whereas in the proposed M4 model, the recommended $\lambda = 0.2$ value is assumed for easy field applicability. To assess model accuracy, four quantitative goodness-of-fit statistics, viz., *RMSE, RSR, NSE* (%), and *MAE* are computed and compared.

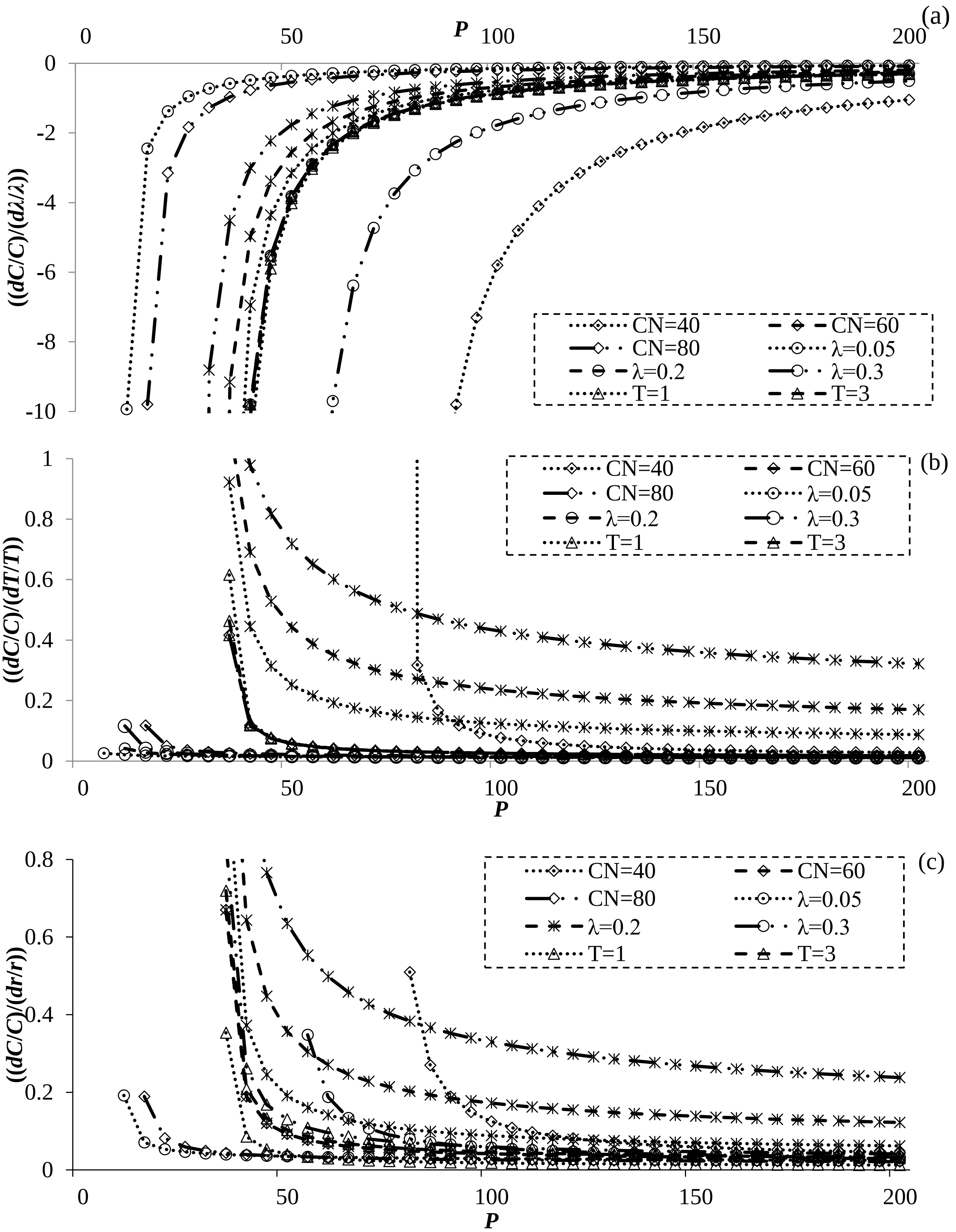


Fig. 3. **Sensitivity analysis of the proposed model w.r.t. its parameters:**
relative change in *C* with respect to partial change in *λ*, *T* and *r*

Fig. 4 (a) illustrates the models' performance based on *RMSE* values, showing the proposed model (M3) having the lowest *RMSE* for all watersheds. On the other hand, Fig. 4 (b) depicts overall *RMSE* for all watersheds under study. It is observed that the mean (median) values for M1, M2, M3, and M4 are 5.49 mm (4.31 mm), 5.73 mm (4.58 mm), 5.16 mm (4.11mm), and 5.50 mm (4.45 mm), respectively, which shows the best performance of the M3 model and the worst performance of the M2 model.

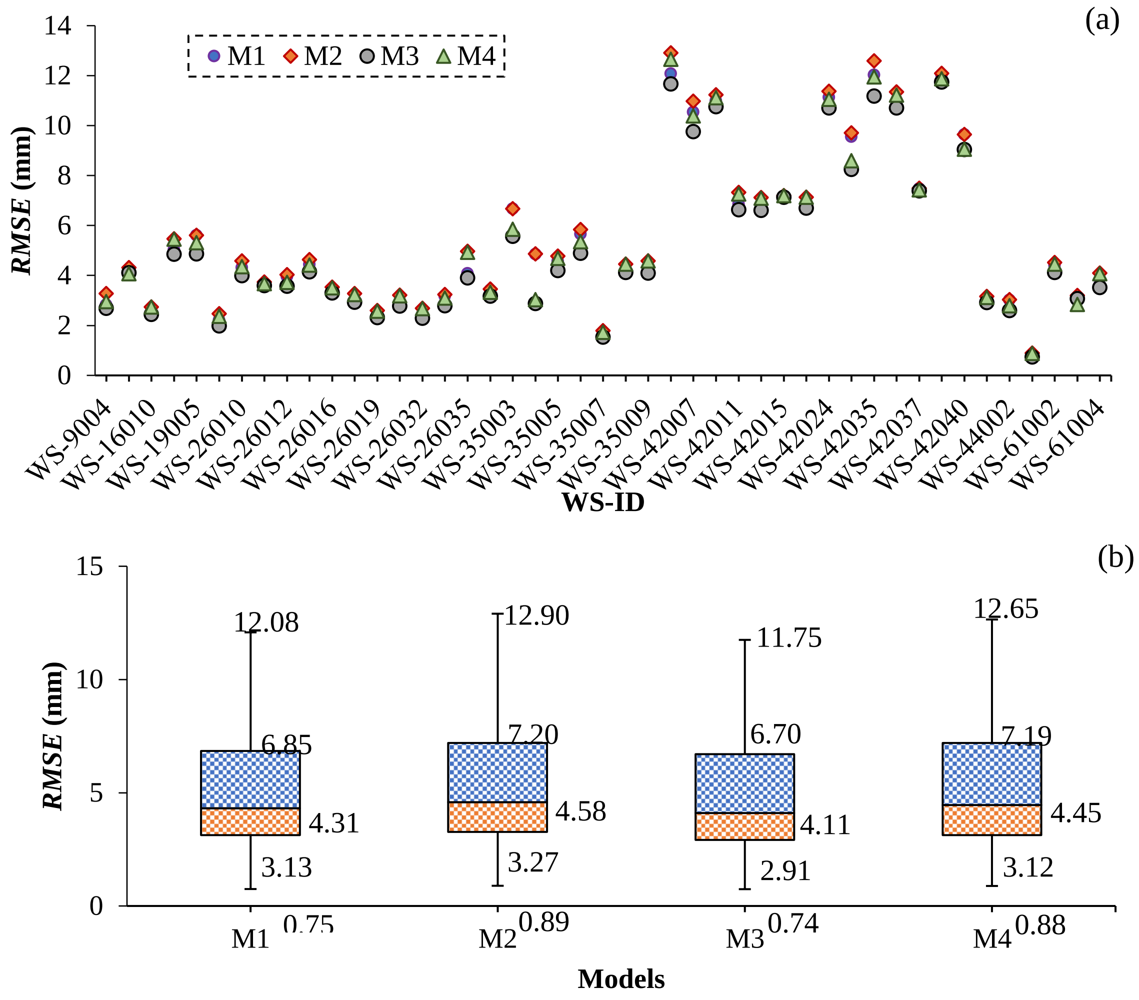
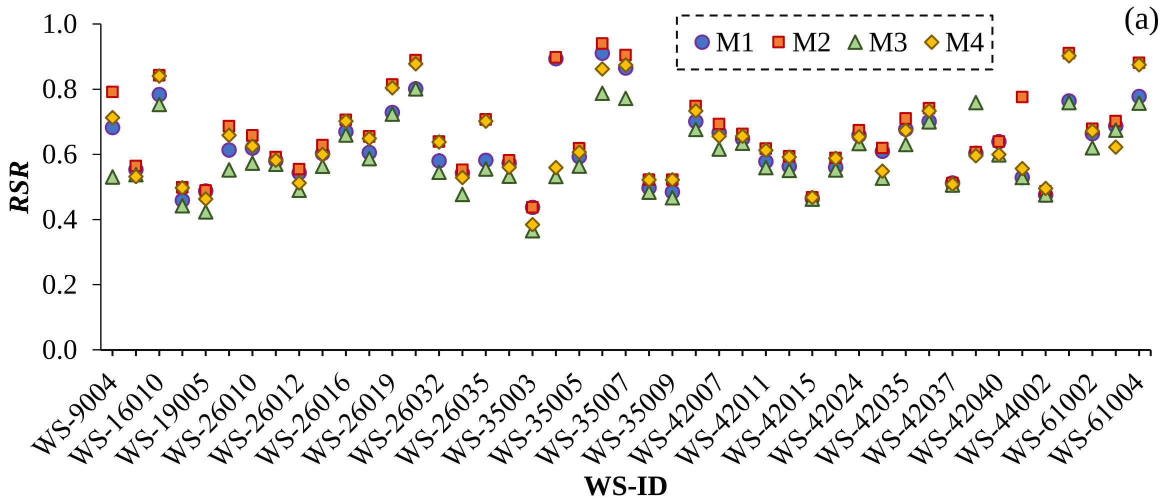


Fig. 4. ***RMSE*** **of all models under study**

Similarly, based on the mean *RSR* value, the proposed model (M3) again depicts improved results in comparison to M1, M2, and M4 models. This improvement is also evident in Fig. 5 (a, b). The Figure shows the overall *RSR* of all watersheds under study, which shows that the proposed model (M3) has the lowest mean (0.59), median (0.56), and inter-quartile range (0.53-0.66) of *RSR*. Based on the *RSR* statistics, the model raking from the best to worst order is given as M3 > M1 > M4 > M2, indicating the M3 model has performed the best.



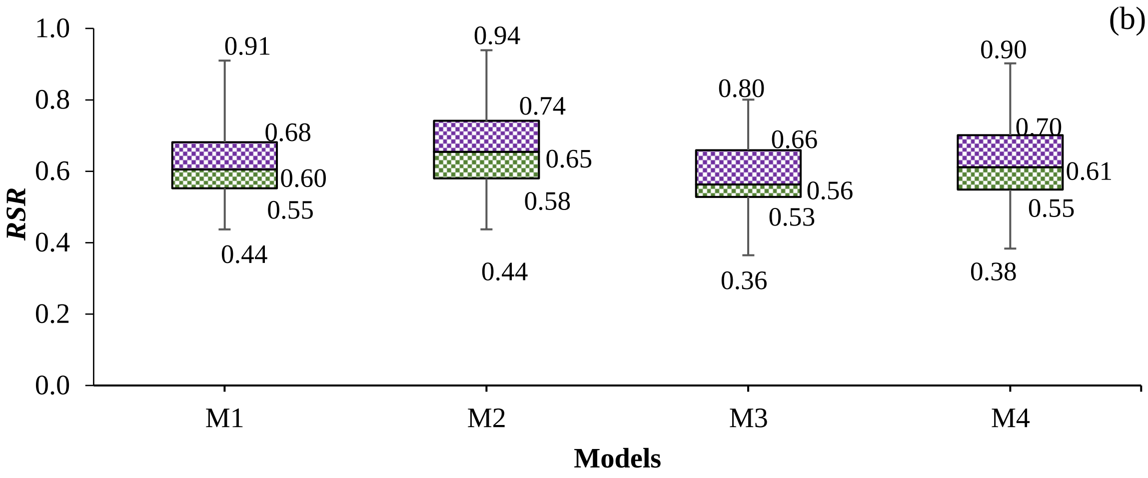


Fig. 5. ***RSR* of all models under study**

Fig. 6 (a, b) compares M1 to M4 models' performance based on the computed *MAE* values. Similar to *RMSE* and *RSR* statistics, M3 has the lowest mean, median, and inter-quartile range of *MAE*, viz., 3.13, 2.34, 1.57-3.81, respectively for all watersheds. To judge the performance based on mean *MAE* values, the model performance in descending order can be arranged as follows: M3 > M1 > M4 > M2.

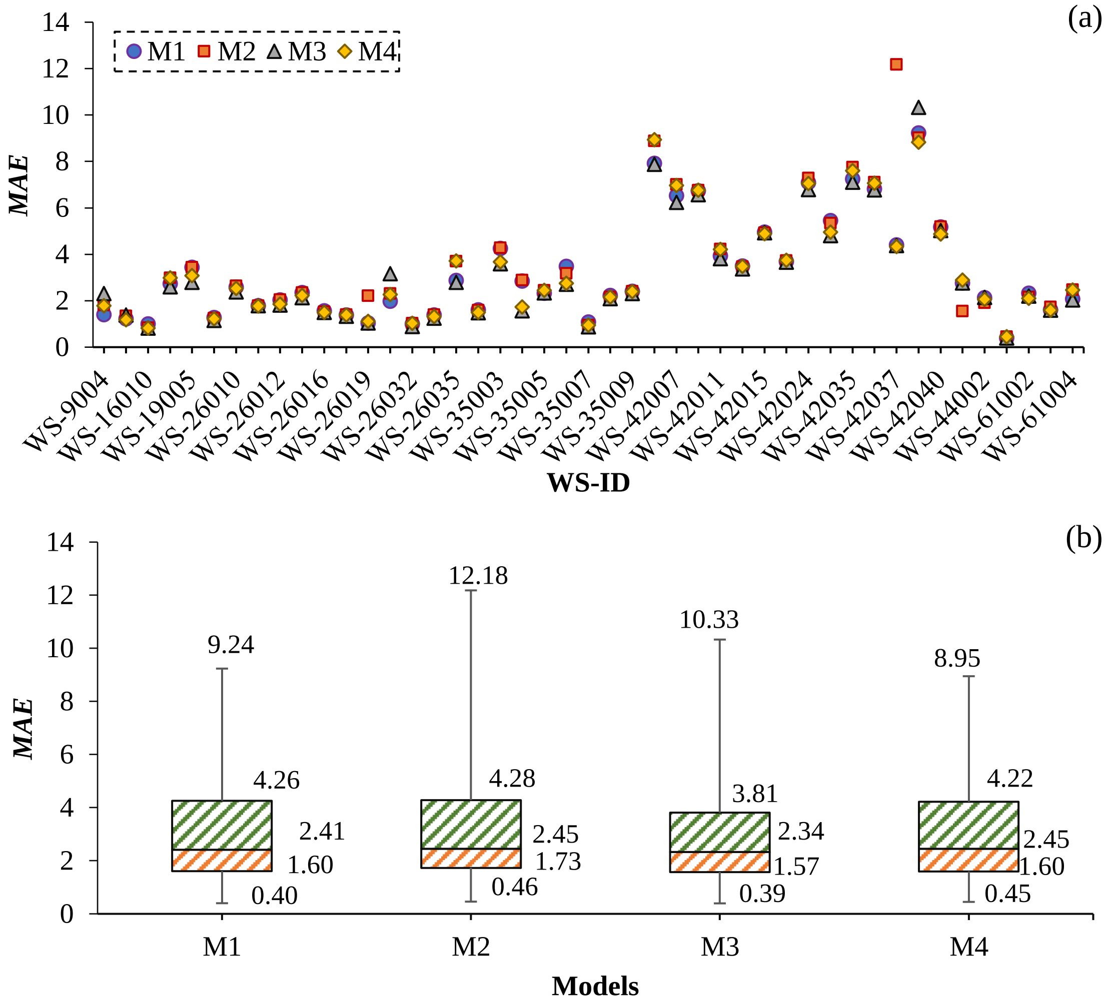


Fig. 5. ***MAE* of all models under study**

Finally, when the models' performance is evaluated on the basis of the fourth performance index, *NSE* (%), the proposed model (M3) is again found to be the best model with highest mean *NSE* (%)

(64.37), whereas the mean *NSE* (%) of M1, M2, and M4 are 59.18, 54.15 and 57.94, respectively. Fig. 7 (a, b) shows *NSE* (%) for all different 45 watersheds and its statistics, indicating that the proposed model M3 shows significant improvement over the other models M1, M2, and M4. The model performance in descending order can be given as follows: M3 > M1 > M4 > M2.

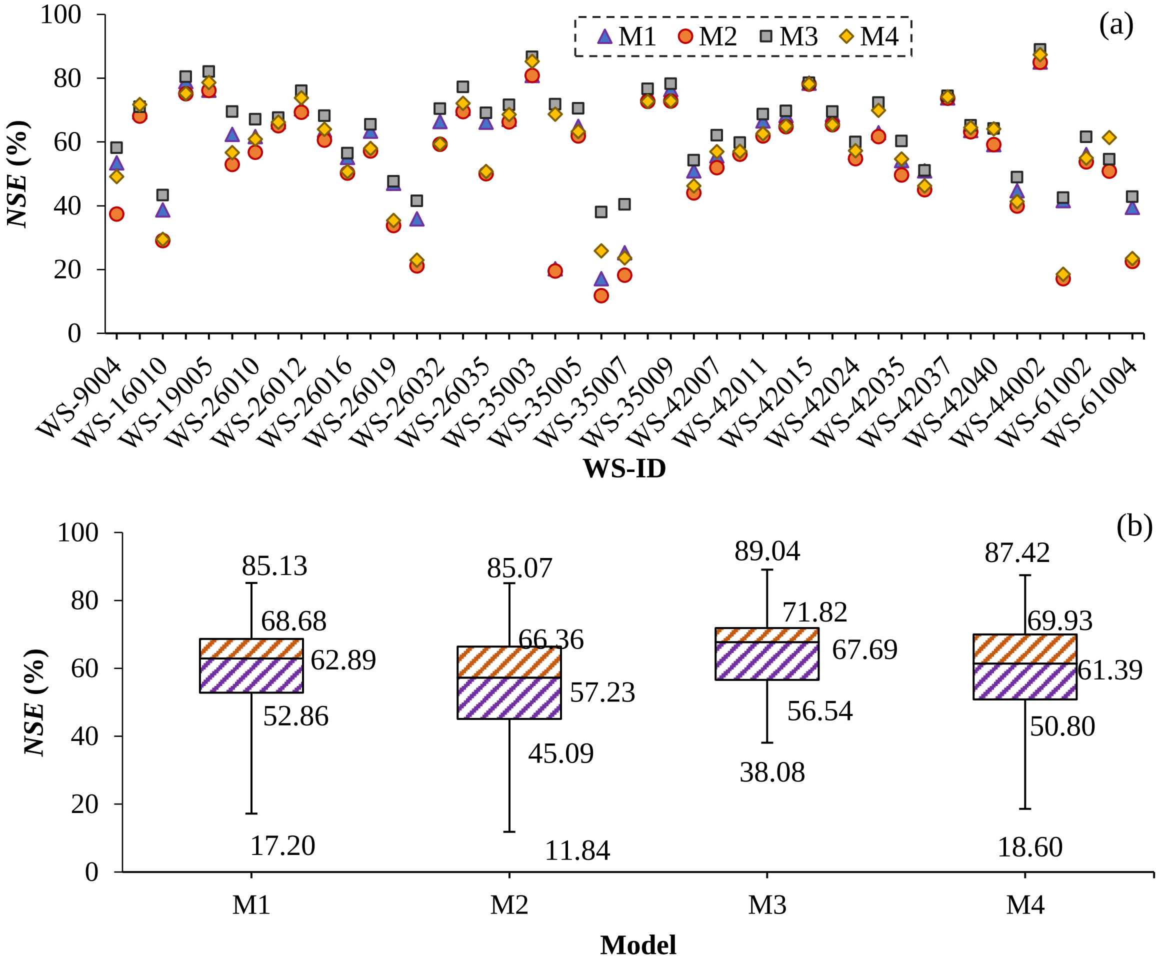


Fig. 7. ***NSE* (%) of all the models under study**

It is because the M2 model was developed based on the assumption (λ = 0.2), which was initially used for field application of the SCS-CN method on the dataset of the USDA-ARS watersheds. Later, the assumed value was criticized by numerous researchers and suggested that the assumption does not have a physical basis. Based on the agreement between observed runoff and computed runoff, the model performance from best to worst is M3 > M1 > M4 > M2.

Overall, the results analysed and visual interpretation through Figures (3) to (6) suggested that the adjusted storm duration incorporated in the proposed approach, i.e., the model M3 should be considered for watershed runoff prediction.

6. Limitations of the study

1. The proposed model does not account for the spatial scale effects.
2. Better results are possible with other values of λ different from the standard value of 0.2 as λ is a region-specific parameter.
3. NEH-4 table cannot be used for the proposed model as the optimized CN values are different from the NEH-4 value for land-soil combinations.

7. Conclusions

In this study, an improved version of the SCS-CN model (M3) is presented incorporating rainfall adjusted based on storm duration which is conceptually more rational for runoff estimation especially in agriculture watersheds. Explicit version of the proposed model (M4) is also more useful for field application. These models are hydrologically more stable and very effective in more accurate runoff prediction. The models' efficacy is tested using the rainfall-runoff data of 45 different US watersheds. The results indicated that the proposed model and its explicit form performed better than the existing models (M1 & M2) with higher values of mean, median and inter-quartile range (64.37, 67.69 and 56.54-71.82 for M3 and 57.94, 61.39 and 50.80-69.93 for M4) of *NSE* (%), respectively. The similar inference is derived when the models are tested using other performance measures, i.e. *RMSE*, *RSR*, and *MAE*. Based on the different performance measures, the models' performance from best to worst can be ranked as M3 > M1 > M4 > M2.

References

[1] Schneider L.E., McCuen R.H. Statistical guidelines for curve number generation. J Irrig Drain Eng. 131, 2005, pp. 282-290.
[2] Verma S., Singh A., Mishra S.K., Singh P.K., Verma R.K. Efficacy of Slope-adjusted Curve Number Models with Varying Abstraction Coefficient for Runoff Estimation, Int J Hydrol Sci Technol, 8(4), 2017, pp. 317-338.
[3] Verma S., Mishra S.K., Verma R.K. Improved runoff curve numbers for large watersheds of the United States, Hydrol Sci J, 2020.DOI: 10.1080/02626667.2020.1832676.
[4] Hawkins R.H. Asymptotic determination of runoff curve numbers from data. J Irrig Drain Eng, ASCE, 119(2), 1993, pp. 334-345.
[5] Soulis K.X., Valiantzas J.D., Dercas N., Londra P.A. Investigation of the direct runoff generation mechanism for the analysis of the SCS-CN method applicability to a partial area experimental watershed, Hydrol Earth Syst Sci, 13(5), 2009, pp. 605-615.
[6] Singh V.P. SCS-CN method revisited using entropy theory, American Society of Agriculture and Biological Engineering, 56(5), 2013, pp. 1805-1820.
[7] Verma S., Verma R.K., Mishra S.K., Singh A., Jayaraj G.K. A Revisit of NRCS-CN Inspired models coupled with RS and GIS for Runoff Estimation. Hydrol Sci J, 62(12), 2017, pp. 1891-1930.
[8] Jain M.K., Mishra S.K., Babu P.S., Venugopal K., Singh V.P. Enhanced runoff curve number model incorporating storm duration and a nonlinear Ia-S relation. J Hydrol Eng 11, 2006, pp. 631-635.
[9] Soulis K.X. Estimation of SCS Curve Number variation following forest fires. Hydrol. Sci J, 63:9, 2018, pp. 1332-1346
[10] Ara Z., Zakwan M. Rainfall runoff modelling for eastern canal basin. Water Energy Int. 61(6), 2018, pp. 63-67.
[11] Moore T.J., Jones M.R. Solving non-linear heat transfer problems using variation of parameters. Int J Thermal Sci, 93, 2015, pp. 29-35.
[12] Barati R. Application of excel solver for parameter estimation of the non-linear Muskingum models. KSCE J Civil Eng 17(5), 2013, pp. 1139-1148.
[13] Verma S., Nema M.K. Development of an empirical model for sub-surface soil moisture estimation and variability assessment in a lesser Himalayan watershed. Model. Earth Syst. Environ, 2021. DOI: 10.1007/s40808-021-01316-z
[14] Zakwan M. Application of optimization techniques to estimate IDF parameters. Water Energy Int. 59(5), 2016, pp. 69-71.
[15] Zakwan M., Muzzammil M., Alam J. Application of spreadsheet to estimate infiltration parameters. Perspect Sci 8, 2016, pp. 702-704.